

Supplementary Material for “Landslide velocity, thickness, and rheology from remote sensing: La Clapière landslide, France”

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1. Numerical approximation of equation (2)

We discretize the conservation of volume equation (equation (2)) on a regular grid using finite difference approximations, such that

$$-\frac{\Delta z_{i,j}}{\Delta t} = u_{i,j} \frac{fh_{i+1,j} - fh_{i-1,j}}{2\Delta x} + v_{i,j} \frac{fh_{i,j+1} - fh_{i,j-1}}{2\Delta y} + fh_{i,j} \left(\frac{u_{i+1,j} - u_{i-1,j}}{2\Delta x} + \frac{v_{i,j+1} - v_{i,j-1}}{2\Delta y} \right), \quad (\text{S1})$$

where the subscripted i and j are indices in x and y , respectively, the Δ symbol denotes the change in the variable it precedes, and u and v are the x - and y -components of the surface velocity field (L T^{-1}), respectively. Since La Clapière lies completely within the finite difference grid, we set the boundary conditions that velocity goes to zero everywhere outside the active landslide. If the landslide did not lie entirely within the grid, a constant velocity or velocity

gradient boundary condition would be more appropriate where the active landslide intersects the boundary. Equation (S1) can be manipulated algebraically to give a system of N linear equations for the N unknown values of fh_{ij} , where N is the number of grid cells being analyzed:

$$-u_{i,j}fh_{i-1,j} + v_{i,j}fh_{i,j+1} + (u_{i+1,j} - u_{i-1,j} + v_{i,j+1} - v_{i,j-1})fh_{i,j} - v_{i,j}fh_{i,j-1} + \quad (S2)$$

$$u_{i,j}fh_{i+1,j} = -2\Delta x \frac{\Delta z_{i,j}}{\Delta t},$$

assuming $\Delta x = \Delta y$. In matrix form, equation (S2) is $Ah_f = b$, where A is a diagonally-dominant matrix consisting of the surface velocities on the left-hand side of equation (S2), $h_f = fh$ is a vector of the unknown thicknesses, scaled by f , and b is a vector of the right-hand side of equation (S2). In this study, we define f as constant, but this assumption could be relaxed to accommodate heterogeneous material properties based on measured vertical velocity profiles or stress-strain rate relationships at different locations within a landslide. Such data would place additional constraints on the inversion and could be important for landslides that incorporate different rock or soil types, are subject to spatial variations in hydrologic conditions, or have strong contrasts in material properties with depth, especially when landslides are deep relative to their length.

The condition number of A for the La Clapière surface velocity data is effectively infinite, so the inversion requires regularization to avoid oscillatory results that are highly unstable to noise in the data.

2. Selection of α and uncertainty estimation

To select appropriate values for the damping parameter α and to estimate the uncertainty of our landslide thickness model, we use a bootstrapping approach in which we downsample all the 1 m resolution data to a 20 m grid, resulting in 400 independent estimates of u_{surf} , $\partial z / \partial t$, and h_f in each larger grid cell. We first calculate $-2\Delta x \Delta z_{i,j} / \Delta t$ (equation (S2)) in each 1 m grid cell,

then determine the standard deviation of these values in each cell of the 20 m grid, which constrains the uncertainty on b to $\sigma_b = 30 \text{ m}^2 \text{ yr}^{-1}$ averaged over the extent of the landslide. The discrepancy principle states that α should be chosen such that the summed squared misfit between the data and model (the first term in expression (3)) should be equal to $N_{ls}\sigma_b^2$, where N_{ls} is the number of data points on the active landslide [Aster *et al.*, 2005, p. 67]. Each of the 400 realizations of the downsampled data gives a slightly different number of grid cells containing the active landslide, which leads to a slightly different value of α , such that $N_{ls} = 2022 \pm 4$, and $\alpha = 0.16 \pm 0.06$ (mean \pm standard deviation). Each downsampling also gives a different estimate of h_f , and we report the mean and standard deviation of h_f in each 20 m grid cell in Figure 2.

3. Effects of α on failure surface smoothness

The discrepancy principle provides a statistically meaningful way to select an ideal α when uncertainties in the elevation change data can be accurately estimated, but knowledge of these uncertainties is not always possible. Figure S1 illustrates how different values of α affect the smoothness of the resulting thickness model when the landslide's volume is held constant by adjusting f . When α is lower than the optimal value (Figure S1a and b) the failure surface is highly oscillatory and overly sensitive to errors in the data, as evidenced by the streaking patterns trending in the direction of landslide motion to the SW. On the other hand, when α is higher than the optimal value, the thickness model is overly smooth such that it does not agree with the measured surface elevation changes within their uncertainties. When uncertainties are unknown, selection of α is more subjective, but can be guided by, for example, the L-curve criterion [Hansen, 1992] or singular value decomposition [Aster *et al.*, 2005, Ch. 4].

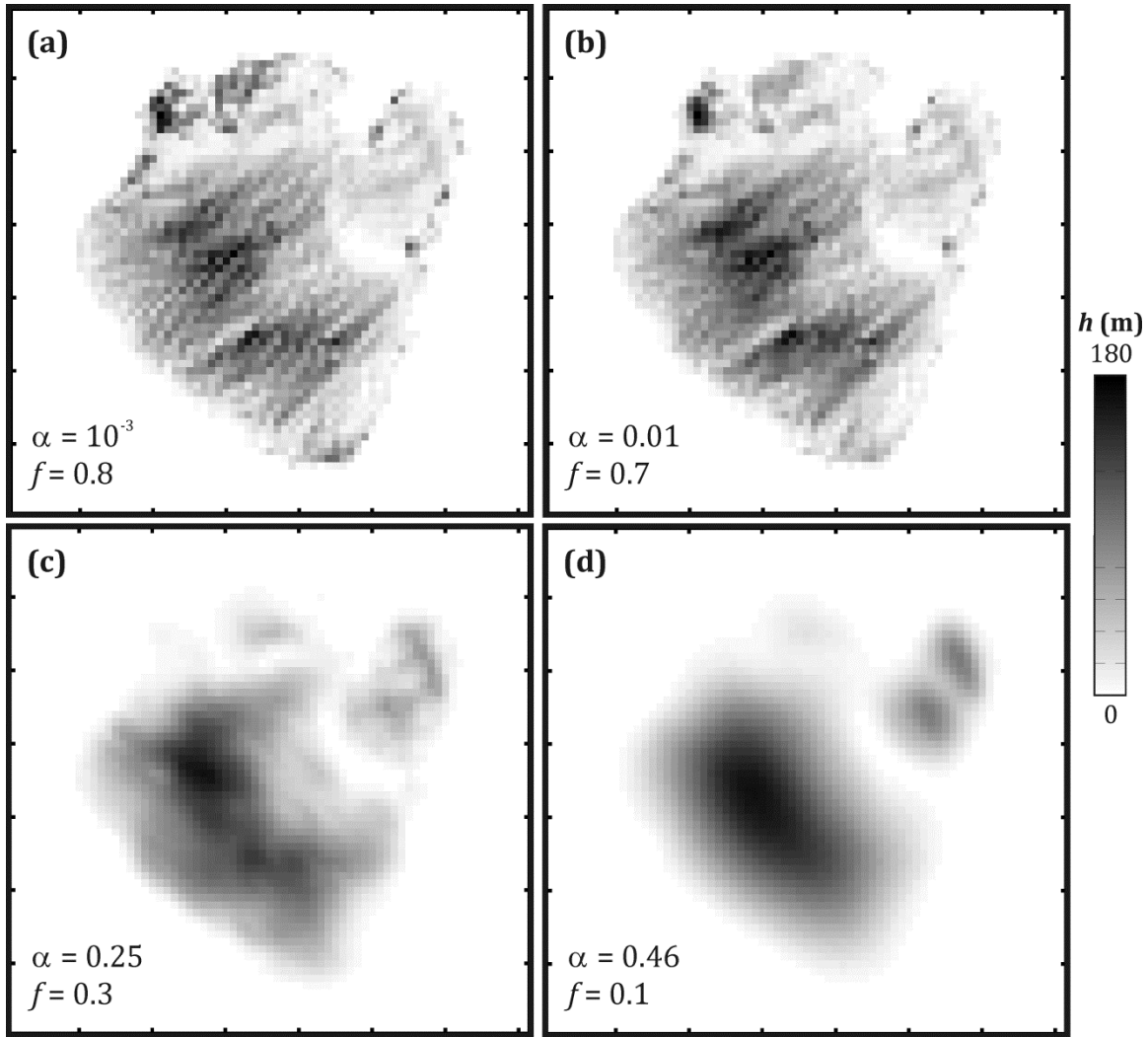


Figure S1. Predicted landslide thickness for selected values of α and f . In all panels, landslide volume is $3.8 \times 10^7 \text{ m}^3$ as in the preferred thickness model, but the thickness model becomes smoother as α increases from below the optimal value (a,b) to above the optimal value (c,d).

References

- Aster, R. C., B. Borchers, and C. H. Thurber (2005), *Parameter Estimation and Inverse Problems*, Elsevier Academic Press, San Diego, CA, United States.
- Hansen, P. C. (1992), Analysis of Discrete Ill-Posed Problems by Means of the L-Curve, *SIAM Review*, 34(4), 561-580, doi:10.1137/1034115.